EXAMPLE 2 Solve a three-variable system with no solution

Solve the system.

$$\begin{aligned} x + y + z &= 3 && \text{Equation 1} \\ 4x + 4y + 4z &= 7 && \text{Equation 2} \\ 3x - y + 2z &= 5 && \text{Equation 3} \end{aligned}$$

REVIEW SYSTEMS
For help with solving linear systems with many solutions or no solution, see p. 160.

Solution

When you multiply Equation 1 by -4 and add the result to Equation 2, you obtain a false equation.

$$\begin{array}{rl} -4x - 4y - 4z = -12 & \text{Add } -4 \text{ times Equation 1} \\ 4x + 4y + 4z = 7 & \text{to Equation 2.} \\ \hline 0 = -5 & \text{New Equation 1} \end{array}$$

▸ Because you obtain a false equation, you can conclude that the original system has no solution.

EXAMPLE 3 Solve a three-variable system with many solutions

Solve the system.

$$\begin{aligned} x + y + z &= 4 && \text{Equation 1} \\ x + y - z &= 4 && \text{Equation 2} \\ 3x + 3y + z &= 12 && \text{Equation 3} \end{aligned}$$

Solution

STEP 1 **Rewrite** the system as a linear system in *two* variables.

$$\begin{array}{rl} x + y + z = 4 & \text{Add Equation 1} \\ x + y - z = 4 & \text{to Equation 2.} \\ \hline 2x + 2y = 8 & \text{New Equation 1} \end{array}$$

$$\begin{array}{rl} x + y - z = 4 & \text{Add Equation 2} \\ 3x + 3y + z = 12 & \text{to Equation 3.} \\ \hline 4x + 4y = 16 & \text{New Equation 2} \end{array}$$

STEP 2 **Solve** the new linear system for both of its variables.

$$\begin{array}{rl} -4x - 4y = -16 & \text{Add } -2 \text{ times new Equation 1} \\ 4x + 4y = 16 & \text{to new Equation 2.} \\ \hline 0 = 0 & \end{array}$$

Because you obtain the identity $0 = 0$, the system has infinitely many solutions.

STEP 3 **Describe** the solutions of the system. One way to do this is to divide new Equation 1 by 2 to get $x + y = 4$, or $y = -x + 4$. Substituting this into original Equation 1 produces $z = 0$. So, any ordered triple of the form $(x, -x + 4, 0)$ is a solution of the system.

GUIDED PRACTICE for Examples 1, 2, and 3

Solve the system.

1. $3x + y - 2z = 10$
$6x - 2y + z = -2$
$x + 4y + 3z = 7$

2. $x + y - z = 2$
$2x + 2y - 2z = 6$
$5x + y - 3z = 8$

3. $x + y + z = 3$
$x + y - z = 3$
$2x + 2y + z = 6$