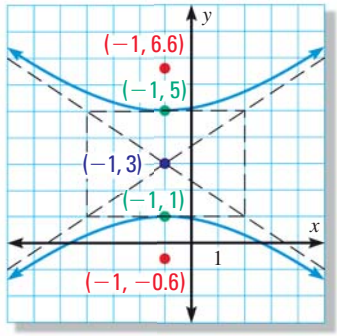
EXAMPLE 2 Graph the equation of a translated hyperbola

Graph $\dfrac{(y-3)^2}{4} - \dfrac{(x+1)^2}{9} = 1$.

Solution

STEP 1 **Compare** the given equation to the standard forms of equations of hyperbolas. The equation's form tells you that the graph is a hyperbola with a vertical transverse axis. The center is at $(h, k) = (-1, 3)$. Because $a^2 = 4$ and $b^2 = 9$, you know that $a = 2$ and $b = 3$.

STEP 2 **Plot** the center, vertices, and foci. The vertices lie $a = 2$ units above and below the center, at $(-1, 5)$ and $(-1, 1)$. Because $c^2 = a^2 + b^2 = 13$, the foci lie $c = \sqrt{13} \approx 3.6$ units above and below the center, at $(-1, 6.6)$ and $(-1, -0.6)$.



SOLVE FOR *y*

To plot additional points on the hyperbola, solve for y to obtain

$$y = 3 \pm 2\sqrt{1 + \frac{(x+1)^2}{9}}.$$

Then make a table of values.

STEP 3 **Draw** the hyperbola. Draw a rectangle centered at $(-1, 3)$ that is $2a = 4$ units high and $2b = 6$ units wide. Draw the asymptotes through the opposite corners of the rectangle. Then draw the hyperbola passing through the vertices and approaching the asymptotes.

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GUIDED PRACTICE for Examples 1 and 2

Graph the equation. Identify the important characteristics of the graph.

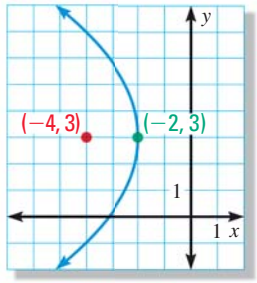
1. $(x+1)^2 + (y-3)^2 = 4$
2. $(x-2)^2 = 8(y+3)$
3. $(x+3)^2 - \dfrac{(y-4)^2}{4} = 1$
4. $\dfrac{(x-2)^2}{16} + \dfrac{(y-1)^2}{9} = 1$

EXAMPLE 3 Write an equation of a translated parabola

Write an equation of the parabola whose vertex is at $(-2, 3)$ and whose focus is at $(-4, 3)$.

Solution

STEP 1 **Determine** the form of the equation. Begin by making a rough sketch of the parabola. Because the focus is to the left of the vertex, the parabola opens to the left, and its equation has the form $(y-k)^2 = 4p(x-h)$ where $p < 0$.



STEP 2 **Identify** h and k. The vertex is at $(-2, 3)$, so $h = -2$ and $k = 3$.

STEP 3 **Find** p. The vertex $(-2, 3)$ and focus $(-4, 3)$ both lie on the line $y = 3$, so the distance between them is $|p| = |-4 - (-2)| = 2$, and thus $p = \pm 2$. Because $p < 0$, it follows that $p = -2$, so $4p = -8$.

▶ The standard form of the equation is $(y-3)^2 = -8(x+2)$.