The Unit Circle

The circle $x^2 + y^2 = 1$, which has center (0, 0)and radius 1, is called the unit circle. The values of $\sin \theta$ and $\cos \theta$ are simply the y-coordinate and x-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle.

$$(x, y)$$
 $r = 1$

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y \qquad \cos \theta = \frac{x}{r} = \frac{x}{1} = x$$

It is convenient to use the unit circle to find trigonometric functions of quadrantal angles. A quadrantal angle is an angle in standard position whose terminal side lies on an axis. The measure of a quadrantal angle is always a multiple of 90°, or $\frac{\pi}{2}$ radians.

EXAMPLE 2

Use the unit circle

Use the unit circle to evaluate the six trigonometric functions of $\theta = 270^{\circ}$.

ANOTHER WAY

The general circle $x^2 + y^2 = r^2$ can also be used to find the trigonometric functions of $\theta = 270^{\circ}$. The terminal side of θ intersects the circle at (0, -r). Therefore:

$$\sin \theta = \frac{y}{r} = \frac{-r}{r} = -1$$

The other functions can be evaluated similarly.

Solution

Draw the unit circle, then draw the angle $\theta = 270^{\circ}$ in standard position. The terminal side of θ intersects the unit circle at (0, -1), so use x = 0 and y = -1 to evaluate the trigonometric functions.

$$\sin \theta = \frac{y}{r} = \frac{-1}{1} = -1$$

$$\sin \theta = \frac{y}{r} = \frac{-1}{1} = -1$$
 $\csc \theta = \frac{r}{y} = \frac{1}{-1} = -1$

$$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$$

$$\sec \theta = \frac{r}{x} = \frac{1}{0}$$
 undefined

$$\tan \theta = \frac{y}{x} = \frac{-y}{0}$$
 undefined $\cot \theta = \frac{x}{y} = \frac{0}{-1} = 0$

$$\cot \theta = \frac{x}{y} = \frac{0}{-1} = 0$$

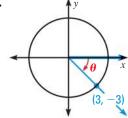


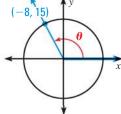
GUIDED PRACTICE

for Examples 1 and 2

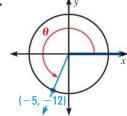
Evaluate the six trigonometric functions of θ .

1.





3.



4. Use the unit circle to evaluate the six trigonometric functions of $\theta = 180^{\circ}$.