KEY CONCEPT

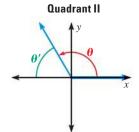
For Your Notebook

Reference Angle Relationships

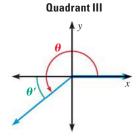
READINGThe symbol θ' is read as "theta prime."

Let θ be an angle in standard position. The **reference angle** for θ is the acute angle θ' formed by the terminal side of θ and the x-axis. The relationship between θ and θ' is shown below for nonquadrantal angles θ such that

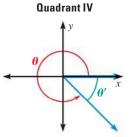
$$90^{\circ} < \theta < 360^{\circ} \left(\frac{\pi}{2} < \theta < 2\pi \right).$$



Degrees: $\theta' = 180^{\circ} - \theta$ Radians: $\theta' = \pi - \theta$



Degrees: $\theta' = \theta - 180^{\circ}$ Radians: $\theta' = \theta - \pi$



Degrees: $\theta' = 360^{\circ} - \theta$ Radians: $\theta' = 2\pi - \theta$

EXAMPLE 3

Find reference angles

Find the reference angle θ' for (a) $\theta = \frac{5\pi}{3}$ and (b) $\theta = -130^{\circ}$.

Solution

- **a.** The terminal side of θ lies in Quadrant IV. So, $\theta' = 2\pi \frac{5\pi}{3} = \frac{\pi}{3}$.
- **b.** Note that θ is coterminal with 230°, whose terminal side lies in Quadrant III. So, $\theta' = 230^{\circ} 180^{\circ} = 50^{\circ}$.

EVALUATING TRIGONOMETRIC FUNCTIONS Reference angles allow you to evaluate a trigonometric function for any angle θ . The sign of the trigonometric function value depends on the quadrant in which θ lies.

For Your Notebook **KEY CONCEPT Evaluating Trigonometric Functions** Use these steps to evaluate a **Signs of Function Values** trigonometric function for any angle θ : Quadrant II ♠ y Quadrant I $\sin \theta$, $\csc \theta$: + $\sin \theta$, $\csc \theta$: + **STEP 1** Find the reference angle θ' . $\cos \theta$, $\sec \theta$: - $\cos \theta$, $\sec \theta$: + **Evaluate** the trigonometric $\tan \theta$, $\cot \theta$: – $\tan \theta$, $\cot \theta$: + function for θ' . Quadrant III Quadrant IV **Determine** the sign of the $\sin \theta$, $\csc \theta$: - $\sin \theta$, $\csc \theta$: trigonometric function value $\cos \theta$, $\sec \theta$: - $\cos \theta$, $\sec \theta$: + from the quadrant in which $\tan \theta$, $\cot \theta$: + $\tan \theta$, $\cot \theta$: – θ lies.