14.6 Apply Sum and Difference Formulas

TEKS a.5, 2A.2.A; P.3.A

Before You found trigonometric functions of a given angle.

Now You will use trigonometric sum and difference formulas.

Why? So you can simplify a ratio used for aerial photography, as in Ex. 43.

Key Vocabulary
- **trigonometric identity,** *p. 924*

In this lesson, you will study formulas that allow you to evaluate trigonometric functions of the sum or difference of two angles.

KEY CONCEPT *For Your Notebook*

Sum and Difference Formulas

Sum Formulas	Difference Formulas
$\sin(a+b) = \sin a\cos b + \cos a\sin b$	$\sin(a-b) = \sin a\cos b - \cos a\sin b$
$\cos(a+b) = \cos a\cos b - \sin a\sin b$	$\cos(a-b) = \cos a\cos b + \sin a\sin b$
$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a\tan b}$	$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a\tan b}$

In general, $\sin(a+b) \neq \sin a + \sin b$. Similar statements can be made for the other trigonometric functions of sums and differences.

EXAMPLE 1 Evaluate a trigonometric expression

Find the exact value of (a) sin 15° and (b) tan $\frac{7\pi}{12}$.

a. $\sin 15° = \sin(60° - 45°)$ — **Substitute 60° − 45° for 15°.**

$= \sin 60°\cos 45° - \cos 60°\sin 45°$ — **Difference formula for sine**

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$ — **Evaluate.**

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ — **Simplify.**

b. $\tan\frac{7\pi}{12} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ — **Substitute $\frac{\pi}{3} + \frac{\pi}{4}$ for $\frac{7\pi}{12}$.**

$= \dfrac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ — **Sum formula for tangent**

$= \dfrac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$ — **Evaluate.**

$= -2 - \sqrt{3}$ — **Simplify.**

REVIEW CONJUGATES

For help with using conjugates to rationalize denominators, see p. 266.